MATH 476/567 ACTUARIAL RISK THEORY FALL 2016
PROFESSOR WANG

Homework 1 (max. points = 100)
Due at the beginning of class on Tuesday, September 13, 2016

You are encouraged to work on these problems in groups of no more than 4. However, each student must hand in her/his own answer sheet. Please show your work enough to show that you understand how to do the problem -and circle your final answer. Full credit can only be given if the answer and approach are appropriate. Please give answers to two decimal places -e.g., xx.xx% and $xx,xxx.xx.

1. The current price of a non-dividend-paying stock is \$40. A European call option on the stock with strike price \$40 and expiration date in three months sells for \$2.78 whereas a 40-strike European put with the same expiration date sells for \$1.99. Find the annual continuously compounded interest rate r. Round your answer to two decimal places.

2. Consider a European call option and a European put option on a non-dividend-paying stock. The current price of the stock is \$45. The call option currently sells for \$0.10 more than the put option. Both the call option and put option will expire in T years. Both the call option and put option have a strike price of \$55. The continuously compounded risk-free interest rate is 3%. Calculate the time of expiration T.

3. A call option on XYZ stock with an exercise price of \$75 and an expiration date one year from now is worth \$5.00 today. A put option on XYZ stock with an exercise price of \$75 and an expiration date one year from now is worth \$2.75 today. The annual effective risk-free rate of return is 8% and XYZ stock pays no dividends. Find the current price of the stock.

4. A stock index currently stands at \$400. The continuously compounded risk-free interest rate is 7%. The dividend yield on the index is 3.5%.
   (1) Find the 4-month forward price.
   (2) Find the prepaid 4-month forward price.

5. Consider different one-year European options, each on ABC stock. Today, you enter into the following portfolio of these one-year options: purchase one 20-strike put for a premium of \$4.00; purchase one 20-strike call for \$5.00; and sell one 30-strike call for \$1.50. Let the price of ABC stock one year from now be 24. The continuously-compounded annual interest rate is 5%. Find the profit or loss from this option portfolio on the expiration date of the options, one year from now. (Consider the time value of money on the option premiums.)

6. A non-dividend-paying stock currently has a price of 108. The continuously-compounded risk-free rate is 6.5%. Assume a one-period binomial pricing model, and let $u = 1.20$ and $d = 0.80$ per year. Consider a one-year European call option with an exercise price of 110. Find the values for Δ and B, and find the premium for this call option. (You must calculate all three values for full credit. Please circle all three values.)

7. Same as problem (6) above, except that, in addition to price changes mentioned above, the stock pays a continuous dividend yield of 1.5%.

(1) Find the values for Δ and B, and find the premium for the call option.
(2) Find the risk-neutral probability of price going up.

8. The price of ABC stock (non-dividend-paying) is currently 100; you hypothesize that, one year from now, the stock will be priced at either 85 or 115. The continuously- compounded rate of interest is 6%. Using the risk-neutral approach, find

(1) The current price of a 105-strike one-year European call option on ABC stock.
(2) The current price of a 90-strike one-year European put option on ABC stock.

9. Consider a European put option on the stock of XYZ with strike \$95 and six months to expiration. XYZ stock does not pay dividends and is currently worth \$100. The annual continuously compounded risk-free interest rate is 8%. In six months the price is expected to be either \$130 or \$80.

(1) Using the single-period binomial option pricing model, find Δ, B and the premium for the put option.
(2) Suppose you observe a put price of \$8. What is the arbitrage? (Describe the strategy and make the cashflow chart)
(3) Suppose you observe a put price of \$6. What is the arbitrage? (Describe the strategy and make the cashflow chart)

10. GS stock is currently worth \$56. Every year, it can increase by 30% or decrease by 10%. The stock pays no dividends, and the annual continuously-compounded risk-free interest rate is 4%. Using a two-period binomial option pricing model, find the price today of one two-year European put option on the stock with a strike price of \$120.

Additional Problems for Math 567 Students (max. points = 20)

Option pricing in risk-averse world

What is the option pricing in the risk-averse world? In this world, we let p denote the real probability of the stock going up. Let α be the continuously compounded expected return on the stock. S is the current stock price and δ is dividend yield. Then p satisfies the equation

$$puSe^{\delta h} + (1-p)dSe^{\delta h} = e^{\alpha h}S.$$

Solving for p we find

$$p = \frac{e^{(\alpha-\delta)h} - d}{u-d}$$

The real probability for the stock to go down is then

$$1-p = \frac{u - e^{(\alpha-\delta)h}}{u-d}.$$

Imposing the condition $d < e^{(\alpha-\delta)h} < u$ we obtain $0 < p < 1$. Now, using p we can find the actual expected payoff at the end of the period:

$$pC_u + (1-p)C_d.$$

For risk-neutral probabilities we discounted the expected payoff at the risk- free rate in order to obtain the current option price. At what rate do we discount the actual expected payoff? Definitely not at the rate α since the option is a type of leveraged investment in the stock so that it is riskier than the stock.

Let γ denote the continuously compounded rate of return of the option, which is also the rate of return for the replicating portfolio consisting of Δ shares of underlying stock and investing \$$B$. Hence we have

$$e^{\gamma h}(S\Delta + B) = S\Delta e^{\alpha h} + Be^{rh}, \quad e^{\gamma h} = \frac{S\Delta e^{\alpha h} + Be^{rh}}{S\Delta + B}.$$

We can now compute the option price as the discounted expected payoff at the rate γ to obtain

$$C_0 = e^{-\gamma h}(pC_u + (1-p)C_d).$$

Remark: Since an option is equivalent to holding a portfolio consisting of buying Δ shares and borrowing \$$B$, the denominator of the previous relation is just the option price. Thus, discounted cash flow is not used in practice to price options: It is necessary to compute the option price in order to compute the correct discount rate.

Questions:

11. Show that the option price obtained with real probabilities is the same as the one with risk-neutral probabilities. [Hint: It is enough to prove $e^{-\gamma h}(pC_u + (1-p)C_d) = S\Delta + B$]

12. Given the following information about a 1-year European call option on a stock: The strike price is $K = 50$, the current price is $S = 51$. In one year the stock price increases by 20% or decreases by 10%. The expected rate of return is 10% and rate of dividend yield is 1%. The continuously compounded risk-free rate is 5%. Use a one-period binomial model to compute the price of the call

(1) Using true probabilities on the stock, and fine γ.
(2) Using risk-neutral probabilities.