

MATH 461 PROBABILITY THEORY SPRING 2017
PROFESSOR WANG

Homework 1 (max. points = 50)
Due at the beginning of class on Monday, January 30, 2017

Each problem is worth 10 points and only **five randomly chosen** problems will be graded. You are encouraged to work on these problems in groups of no more than 4. However, each student must hand in her/his own answer sheet. Please show your work enough to show that you understand how to do the problem -and circle your final answer. Full credit can only be given if the answer and approach are appropriate.

Chapter 2. 2, 3, 5

Chapter 1. 5, 8, 11

Additional Problems.

1. (a) How many vectors $(x_1, x_2, x_3, \dots, x_n)$ are there for which each x_i is either 0 or 1 and

$$x_1 + x_2 + \dots + x_n = k.$$

- (b) Do the same problem as before but under the condition that

$$x_1 + x_2 + \dots + x_n \geq k.$$

2. In how many ways can n identical balls be distributed into k bins such that each bin contains at least two balls. Assume that $n \geq 2k$.

3. A die is rolled until either 4 or 5 appears. Find the probability that a 5 occurs first. Simplify the answer.

Hint: Let E_n denote the event that a 5 occurs on the n -th roll and no 4 or 5 occurs on the first $n-1$ rolls. Find $P(E_n)$ and express the above probability in terms of them.

4. A card player is dealt a 13 card hand from a well-shuffled, standard deck of cards. What is the probability that the hand is void in at least one suit (“void in a suit” means having no cards of that suit)?

Hint: Let E_i be the event that the hand is void in the suit i for $i = 1, 2, 3, 4$ (*clubs, hearts, diamonds* and *spades*).