Lecture 5: Homogeneous eqn's.

Consider $\frac{dy}{dx} = \frac{y-4x}{x-y}$.

1. It can be written as

$$\frac{dy}{dx} = \frac{\frac{y}{x} - 4}{1 - \frac{y}{x}} \quad \textcircled{+}$$

↓ divide both up/down sides by x.

2. Introduce v. s.t. $v = \frac{y}{x}$ or $y = xv(x)$

write $\frac{dy}{dx} = \frac{d(xv(x))}{dx} = v(x) + xv'$

plug in ⊙.

$$v(x) + xv' = \frac{v-4}{1-v}.$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v-4}{1-v} - v = \frac{v^2-4}{1-v}$$

it's solvable. ☺

3. solve the solvable ODE

$$\frac{(1-v)\,dv}{v^2-4} = \frac{dx}{x}$$

$$\frac{dv}{v^2-4} - \frac{v\,dv}{v^2-4} = \frac{dx}{x}.$$

$$\frac{1}{v^2-4} = \left(\frac{1}{v-2} - \frac{1}{v+2}\right)\cdot\frac{1}{4}.$$

$$\frac{1}{4}\ln\left|\frac{v-2}{v+2}\right| - \frac{1}{2}\ln|v^2-4| = \ln|x| + C$$

$$\Rightarrow \ln\frac{|v-2|}{|v+2|}\cdot\frac{1}{|v^2-4|^2} = \ln|x|^4 + C \Rightarrow \frac{C}{|v+2|^3\,|v-2|} = |x|^4$$

$$\left|\frac{y}{x}+2\right|^3\left|\frac{y}{x}-2\right| = \frac{c}{x^4}$$

$$\Rightarrow \quad |y+2x|^3 \cdot |y-2x| = c$$

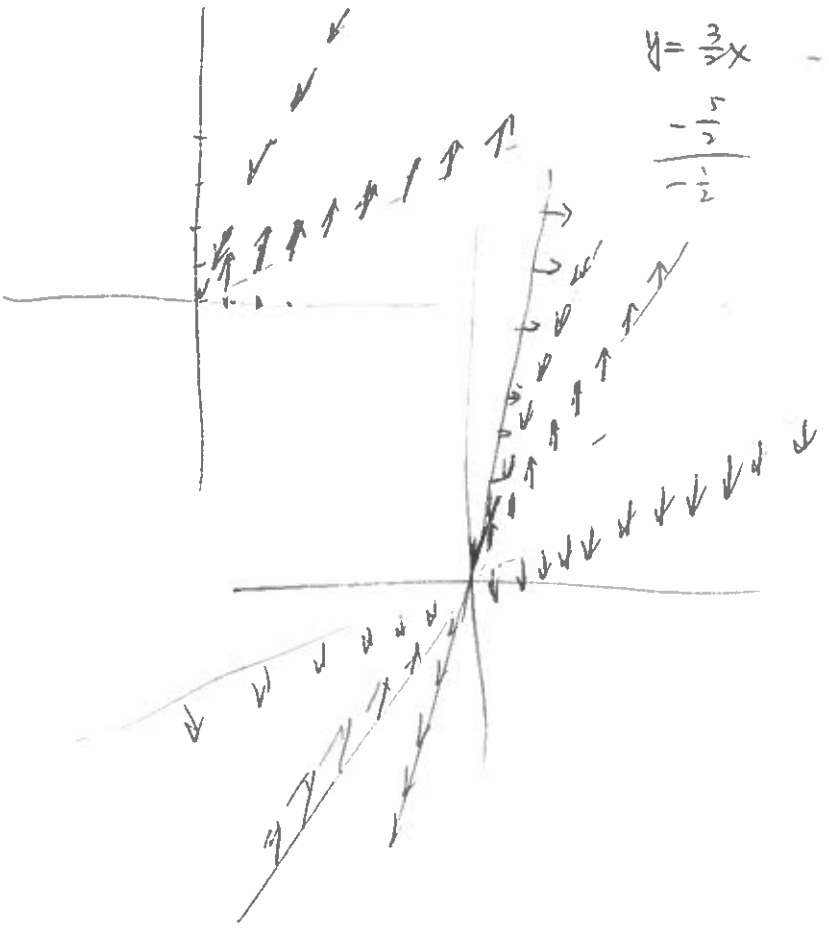
$$\Rightarrow \quad (y+2x)^3 |y-2x| = c.$$

Slope field of $\frac{dy}{dx} = \frac{y-4x}{x-y}$.

$y = 2x$ $\quad y=2,\ x=1$ $\quad y=4,\ x=2$ $\quad \frac{-2x}{x-2x} = -2$

$y = \frac{1}{2}x$ $\quad x=2,\ y=1$ $\quad \frac{2-4}{2-1} = -2$

$y = \frac{3}{2}x$ $\quad \frac{-\frac{5}{2}}{-\frac{1}{2}} = 5$



* direction field and integral curves are symm. w.r.t the origin.

General homogeneous eqn:

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right).$$

Substitution $y = xv(x)$.

$$\frac{dy}{dx} = v(x) + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = f(v) \quad \Rightarrow \quad \frac{dv}{f(v)-v} = \frac{dx}{x}$$

Seperable. eqn. can integrate directly.

Hmk: Sec 2.2. # 33. 36.

practice # 34. 37. 38.