Math 441 ODE

HW# 8

3.5. 7, 12, 20
3.6. 9, 14

3.5.7. $y''+9y=t^2e^{3t}+6$ 20

1) $y''+9y=0$. $r^2+9r=0 \Rightarrow r=\pm 3i$.
$\Rightarrow y = c_1\cos 3x + c_2 \sin 3x$

2) Particular solution:
Assume $y=a_1t^2e^{3t}+a_2te^{3t}+a_3e^{3t}+a_4$.
$y''+9y=t^2e^{3t}+6$

$$\Rightarrow \begin{cases} 18a_1=1 \\ 12a_1+18a_2=0 \\ 2a_1+6a_2+18a_3=0 \\ 9a_4=6 \end{cases}$$

$$\Rightarrow \begin{cases} a_1=1/18 \\ a_2=-1/27 \\ a_3=1/162 \\ a_4=2/3 \end{cases}$$

$$\Rightarrow y = c_1\cos 3x + c_2\sin 3x + \frac{1}{18}t^2e^{3t}-\frac{1}{27}te^{3t}+\frac{1}{162}e^{3t}+\frac{2}{3}$$

$$= c_1\cos 3x + c_2 \sin 3x + \frac{1}{162}(9t^2-6t+1)e^{3t}+\frac{2}{3}.$$

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12. $u'' + w_0^2 u = \cos w_0 t$.

1) $u'' + w_0^2 u = 0 \Rightarrow u = c_1\cos(w_0 t) + c_2 \sin(w_0 t)$.

2) Assume $u = c_1 t\cos(w_0 t) + c_2 t\sin(wt)$.

$u'' + w_0^2 u = \cos w_0 t$

$\Leftrightarrow -2c_1 w_0 \sin(w_0 t) + 2c_2 w_0 \cos(w_0 t)$

$= \cos w_0 t$

$\Rightarrow c_1 = 0,\ c_2 = \frac{1}{2w_0}$

$\Rightarrow u = c_1\cos(w_0 t) + c_2\sin(w_0 t) + \frac{1}{2w_0} t\sin(w_0 t)$.

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20. $y'' + 2y' + 5y = 4e^{-t}\cos 2t$. $y(0) = 1$, $y'(0) = 0$.

1) $y'' + 2y' + 5y = 0$.

$r^2 + 2r + 5 = 0$.

$r = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$.

$\Rightarrow y = c_1 e^{-t}\cos 2t + c_2 e^{-t}\sin 2t$.

2). $y'' + 2y' + 5y = 4e^{-t}\cos 2t = \text{Re}(4e^{-t}e^{2it})$

Solve $y'' + 2y' + 5y = 4e^{(-1+2i)t}$ first.

Assume $Y(t) = a t e^{(-1+2i)t}$.

then

$y'' + 2y' + 5y = 4a_1 i e^{(-1+2i)t} = 4e^{(-1+2i)t}$

$\Rightarrow a_1 = -i$.

The solution $y(t) = \text{Re}(-ite^{(-1+2i)t})$.

$= te^{-t}\sin 2t$

$\Rightarrow Y(t) = c_1 e^{-t}\cos 2t + c_2 e^{-t}\sin 2t + te^{-t}\sin 2t$

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$y(0)=1 \Rightarrow c_1 = 0.$ $y'(0)=0 \Rightarrow c_2 = \frac{1}{2}$

$\Rightarrow Y(t) = e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t + te^{-t}\sin 2t$

3.6.9. $4y''+y = 2\sec(t/2)$. 20

1). $4y''+y=0$.

$y = c_1\cos(t/2) + c_2\sin(t/2)$.

2. By variation of parameter, Theorem 3.6.1, we know a particular solution is

$Y(t) = t\sin\left(\frac{t}{2}\right) + 2\cos\frac{t}{2}\ln\cos\left(\frac{t}{2}\right)$

$\Rightarrow$ general solution is.

$Y(t) = c_1\cos\left(\frac{t}{2}\right) + c_2\sin\left(\frac{t}{2}\right) + t\sin\left(\frac{t}{2}\right) + 2\cos\frac{t}{2}\ln\left(\cos\frac{t}{2}\right)$

3.6.14. $t^2 y'' - t(t+2)y' + (t+2)y = 2t^3, \; t > 0.$

$y_1 = t, \; y_2(t) = te^t.$

20.

Sol:

$t^2 y_1'' - t(t+2)y_1' + (t+2)y_1$

$= 0 - t(t+2) + (t+2)t = 0.$

$t^2 y_2'' - t(t+2)y_2' + (t+2)y_2$

$= 2t^2e^t + t^3e^t - t^2e^t - t^3e^t - 2te^t$

$-2t^2e^t + t^2e^t + 2te^t = 0.$

$$y'' - \left(1 + \frac{2}{t}\right)y' + \frac{t+2}{t^2}y = 2t.$$

$$W(y_1, y_2)(t) = \begin{vmatrix} t & te^t \\ 1 & e^t + te^t \end{vmatrix} = t^2e^t$$

$$Y(t) = -t\int \frac{se^s \, 2s}{s^2e^s} ds$$

$$+ te^t \int \frac{s \, 2s}{s^2e^s} ds$$

$$= -2t\int ds + te^t 2\int e^{-s} ds.$$

$$= -2t(t \quad) + 2te^t(\quad -e^{-t})$$

$$= -2t^2 - 2t.$$

~~choose $t_0 = 1$, then~~

~~$Y(t) = -2t^2 + 2te^{(t-1)}$~~