Math 441 HW# 6

3.1. 15: $y''+8y'-9y=0,\ y(1)=1,\ y'(1)=0.$

$r^2+8r-9=0 \iff (r+9)(r-1)=0.$

$\iff r_1=-9$ and $r_2=1.$

$\Rightarrow y = c_1e^{-9t}+c_2e^{t}.$

15

$y(1)=1 \Rightarrow c_1e^{-9}+c_2e=1$

$y'(1)=0 \Rightarrow -9c_1e^{-9}+c_2e=0.$

$\Rightarrow c_1=e^9/10 \quad c_2=9/(10e).$

$\therefore\ y=\frac{e^9}{10}e^{-9t}+\frac{9}{10e}e^{t}$

18. $y_\phi = c_1e^{-t/2}+c_2e^{-2t}.$

$(r+1/2)(r+2)=0$

$r^2+2.5r+1=0.$

$y''+2.5y'+y=0.$

$2y''+5y'+2y=0.$

15

21. $y''-y'-2y=0. \quad r^2-r-2=0 \iff (r-2)(r+1)=0.$

$y=c_1e^{2t}+c_2e^{-t}.$

$y(0)=\alpha \Rightarrow c_1+c_2=\alpha,$

$y'(0)=2 \Rightarrow 2c_1-c_2=2.$

15.

$\Rightarrow c_1=\frac{2+\alpha}{3},\ c_2=\frac{2\alpha-2}{3}$

$\Rightarrow y=\frac{2+\alpha}{3}e^{2t}+\frac{2\alpha-2}{3}e^{-t}.$

$t\to\infty,\ y\to 0$, then $\alpha=-2$. otherwise, $y\to\pm\infty$.

10

3.2. 5. $\begin{vmatrix} e^t\sin t & e^t\cos t \\ e^t\sin t + te^t\cos t & e^t\cos t - e^t\sin t \end{vmatrix}$

$= e^{2t}\sin t\cos t - e^{2t}\sin^2 t - e^{2t}\sin t\cos t$

$-e^{2t}\cos^2 t = -e^{2t}$.

15

8. $(t-1)y'' - 3ty' + 4y = \sin t$, $y(-2) = 2$, $y'(-2) = 1$

Need

$t-1 \neq 0$, $t \neq -1$.

$\mathbb{R}\setminus\{-1\} = (-\infty, -1) \cup (-1, \infty)$

$-2 \in (-\infty, -1) \Rightarrow (-\infty, -1)$ is the interval.

15

18. $\begin{vmatrix} t & g(t) \\ 1 & g'(t) \end{vmatrix} = t^2e^t \Rightarrow tg'(t) - g(t) = t^2e^t$.

$\cancel{\Rightarrow (tg(t))' = t^2e^t + C}$.

$\cancel{\Rightarrow tg(t) = t^2e^t - 2te^t + 2e^t + C}$.

$\cancel{\Rightarrow g(t) = te^t - 2e^t + 2e^t/t + C/t}$.

$\frac{1}{t}g'(t) - \frac{1}{t^2}g(t) = e^t$

$\Rightarrow \left(\frac{1}{t}g(t)\right)' = e^t \Rightarrow \frac{1}{t}g(t) = e^t + C$

$\Rightarrow g(t) = te^t + ct$.

24. $y''+4y=0$. $y_1(t)=\cos 2t$, $y_2(t)=\sin 2t$.

$y_1''(t) = -4\cos 2t \Rightarrow y_1''(t)+4y_1(t)=0$

similarly, $y_2''(t)+4y_2(t)=0$.

To determine whether they are linearly independent, we write.

$$\begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix}$$

15.

$= 2\cos^2 2t + 2\sin^2 2t = 2 \neq 0$

$\Rightarrow$ these two solutions are independent and hence form a ~~solution set~~ fundamental set of solutions.