Math 441 HW#5.

2.3.3. $\frac{dQ_1}{dt} = 1 - \frac{Q_1}{50}$, phase 1

$\Rightarrow Q_1(t) = 50 - 50e^{-t/50}$ $(Q(0)=0)$.

Phase 2:

$\frac{dQ}{dt} = \frac{-Q}{50} \Rightarrow Q = Ce^{-t/50}$

$Q(0) = Q_1(10) \Rightarrow Q(t) = (50-50e^{-1/5})e^{-t/50}$

$\Rightarrow Q(10) = (50-50e^{-1/5})e^{-1/5} = 7.42$.

7. $\frac{dS}{dt} = rS \Rightarrow S = Ce^{rt} = S_0 e^{rt}$.

a. $2S_0 = S_0 e^{rt} \Rightarrow e^{rt} = 2 \Rightarrow rt = \ln 2$.

b. $T = \ln 2 / (0.07) = 9.9$.

c. $r = \frac{\ln 2}{8} = 0.08664$.

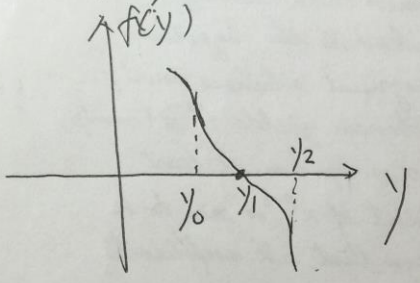
8. $\frac{dS}{dt} = rS + k \Rightarrow S = \frac{k}{r}(e^{rt} - 1)$ $(S(0)=0)$.

b. $10^6 = \frac{k}{0.075}(e^{0.075 \cdot 40} - 1) \Rightarrow k = 3929$.

c. $10^6 = \frac{2000}{r}(e^{40r} - 1) \xrightarrow[\text{computer.}]{\text{by}} r = 9.77\%$

14. $\frac{dy}{dt} = f(y)$, $f(y_1) = 0$

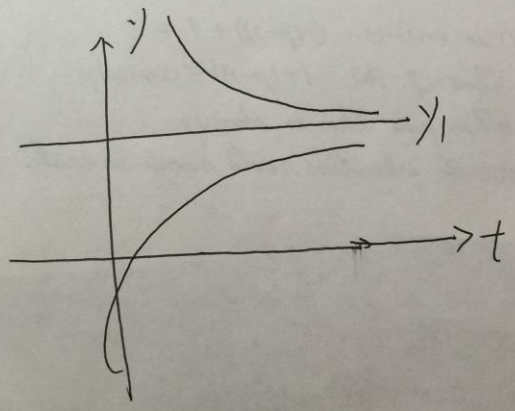
& $f'(y_1) < 0$



then for some $y_0 < y_1$,
$f(y)$ is positive
for $y \in (y_0, y_1)$.

$\Rightarrow \frac{dy}{dt} = f(y) > 0$

$\Rightarrow$ y increases $\forall t$.



for $y_2 > y_1$,
$y \in (y_1, y_2)$

$\frac{dy}{dt} = f(y) < 0$

$\Rightarrow$ y decrease $\forall t$

Similarly.

2.5. 15] $\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right)$.

(a). $y = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}$

$y_0 = \frac{K}{3}$, $\Rightarrow y = \frac{K}{1+2e^{-rt}}$,

If

$y = 2y_0$ then $\frac{K}{1+2e^{-rt}} = \frac{2K}{3}$

$\Rightarrow e^{rt} = 4 \Rightarrow rt = \ln 4$.

$r = 0.025 \Rightarrow t = 55.45$.

(b). $y = \frac{2K^2}{2K + K(1-2)e^{-rt}} = \beta K$

$\Rightarrow 2 = \beta 2 + \beta(1-2)e^{-rt}$

$\Rightarrow e^{rt} = \frac{\beta(1-2)}{2(1-\beta)}$

$\Rightarrow rt = \ln \frac{\beta(1-2)}{2(1-\beta)}$.

$\therefore T = 175$.