Math 441 HW#2.

$\boxed{2.2}$ 5.

$$y' = (\cos^2 x)(\cos^2 2y).$$

1) If $\cos^2 2y \neq 0$, then

$$\frac{dy}{\cos^2 2y} = \cos^2 x \, dx$$

$$\Leftrightarrow \sec^2 2y \, dy = \frac{1+\cos 2x}{2} dx$$

$$\Leftrightarrow \frac{1}{2}\tan 2y = \frac{x + (\sin 2x)/2}{2} + C$$

$$\Leftrightarrow 2\tan 2y - 2x - \sin 2x = C.$$

2) $\cos^2 2y = 0$

$$\Leftrightarrow y = \pm (2n+1)/4 \, \pi, \; n \in \mathbb{Z}.$$

21. $y' = (1+3x^2)/(3y^2 - 6y)$.

$$\Leftrightarrow (3y^2 - 6y)dy = (1+3x^2)dx.$$

$$\Leftrightarrow y^3 - 3y^2 - x - x^3 + C = 0$$

$$y(0) = 1 \Rightarrow C = 2 \Rightarrow y^3 - 3y^2 - x - x^3 + 2 = 0. \quad (1)$$

$y' = \dfrac{1+3x^2}{3(y^2 - 2y)}$ is undefined at $y=0$ & $y=2$.
(vertical tangent).

When $y = 0$, $(1) \Leftrightarrow x^3 + x - 2 = 0$

$$\Leftrightarrow (x-1)(x^2 + x + 2) = 0$$

$$\Leftrightarrow x = 1.$$

When $y=2$, $(1) \Leftrightarrow x^3+x+2=0$

$$\Leftrightarrow (x+1)(x^2-x+2)=0$$

$$\Leftrightarrow x=-1.$$

$\Rightarrow$ The integral curve has vertical tangent at $x=\pm 1$.

$\mathbb{R} \setminus \{\pm 1\} = (-\infty; -1) \cup (-1, 1) \cup (1, \infty)$.

$0 \in (-1, 1)$ (the initial condition)

$\Rightarrow (-1,1)$ is the interval where the solution is valid.

$\boxed{2.6}$ 7. $(e^x \sin y - 2y \sin x) + (e^x \cos y + 2\cos x) y' = 0$

$\Rightarrow (e^x \sin y - 2y\sin x)dx + (e^x \cos y + 2\cos x)dy = 0$

$$\frac{\partial}{\partial y}(e^x \sin y - 2y \sin x) = e^x \cos y - 2y \sin x$$

$$= \frac{\partial}{\partial x}(e^x \cos y + 2\cos x).$$

$\Rightarrow$ it is exact.

$$\int (e^x \sin y - 2y\sin x)\,dx = e^x \sin y + 2y\cos x + h(y).$$

$$\frac{\partial}{\partial y}(e^x \sin y + 2y\cos x + h(y)) = e^x \cos y + 2\cos x$$

$\Rightarrow h'(y)) = 0$

$\Rightarrow$ solution is $e^x \sin y + 2y\cos x + C = 0$

14. $(9x^2+y-1)-(4y-x)y'=0.$

$$\frac{\partial}{\partial y}(9x^2+y-1)=1=\frac{\partial}{\partial x}(-4y+x)$$

$\Rightarrow$ it's exact.

$$\int(9x^2+y-1)\,dx = 3x^3+xy-x+h(y)$$

$$\frac{\partial}{\partial y}(3x^3+xy-x+h(y)) = (-4y+x)$$

$\Leftrightarrow h'(y) = -4y \Leftrightarrow h(y) = -2y^2.$

$\Rightarrow 2y^2-xy+x-3x^3+C=0.\quad (y(1)=0)$

$\Rightarrow 2y^2-xy+x-3x^3+2=0$

$$\Rightarrow y = \frac{x \pm \sqrt{24x^3+x^2-8x-16}}{4}$$

$y(1)=0$

$$\Rightarrow y = \frac{x - \sqrt{24x^3+x^2-8x-16}}{4}.$$

The solution is valid when

$24x^3+x^2-8x-16>0$

which is approximately $x>1$.

15. $(xy^2+bx^2y)+((x+y))x^2y'=0$

$\frac{\partial}{\partial y}(xy^2+bx^2y) = 2xy+bx^2$

$= \frac{\partial}{\partial x}(x+y)x^2 = (3x^2+2xy).$

$\Leftrightarrow b=3.$

$\int(xy^2+3x^2y)dx = \frac{x^2y^2}{2}+x^3y+h(y)$

$h(y)=0 \Rightarrow x^2y^2+2x^3y=C.$

22. $(x+2)\sin y+(x\cos y)y'=0.\ \mu(x,y)=xe^x.$

$\frac{\partial}{\partial y}(x+2)\sin y=(x+2)\cos y \neq \frac{\partial}{\partial x}(x\cos y)$

$\Rightarrow$ not exact.

$\frac{\partial}{\partial y}(xe^x(x+2)\sin y) = xe^x(x+2)\cos y$

$= \frac{\partial}{\partial x}(x^2e^x\cos y) = xe^x(x+2)\cos y$

$\Rightarrow$ it's exact now.

$\int xe^x(x+2)\sin y\,dx = \int(x^2+2x)e^x\sin y\,dx$

$= x^2e^x\sin y+h(y).\quad h(y)=0$

$\Rightarrow x^2e^x\sin y = C.$

27)

$$1+\left(-\frac{x}{y}-\sin y\right)y'=0.$$

$$\frac{d\mu}{dy}=\frac{N_x-M_y}{M}\mu$$

$$=\frac{1/y}{1}$$

$$=\frac{\mu}{y}.$$

$$\Rightarrow \mu=y.$$

$$y+(x-y\sin y)y'=0.$$

$$\Rightarrow xy+y\cos y-\sin y=C.$$