Math 441 ODE

HW #10.

7.1, 4, 5, 4.4, 7, 4.3 5, 18.

7.1.4 $u^{(4)} - u = 0. \iff u^{(4)} = u.$

$x_1 = u,\ x_2 = u',\ x_3 = u'',\ x_4 = u'''.$

Then

$x_1' = x_2,\ x_2' = x_3,\ x_3' = x_4,\ x_4' = x_1.$

5. $u'' + 0.25u' + 4u = 2\cos 3t.\quad u(0) = 1, u'(0) = -2.$

$x_1 = u,\ x_2 = u'.$

Then $x_1' = x_2,$

$x_2' = 2\cos 3t - 4x_1 - 0.25x_2.$

$x_1(0) = 1,\ x_2(0) = -2.$

4.4, 7 $y''' - y'' + y' - y = \sec t.$

$r^3 - r^2 + r - 1 = 0 \iff (r^2+1)(r-1) = 0$

$\Rightarrow r_1 = i,\ r_2 = -i,\ r_3 = 1.$

$y_1 = e^t,\ y_2 = \cos t,\ y_3 = \sin t.$

$$W = \begin{vmatrix} e^t & \cos t & \sin t \\ e^t & -\sin t & \cos t \\ e^t & -\cos t & -\sin t \end{vmatrix} = e^t(\sin^2 t + \cos^2 t) - \cos t(-e^t\sin t - e^t\cos t) + \sin t(-e^t\cos t + e^t\sin t).$$

$= e^t + e^t = 2e^t.$

$$W_1(t) = \begin{vmatrix} 0 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ 1 & -\cos t & -\sin t \end{vmatrix} = 1,\quad W_2(t) = -e^t\cos t + e^t\sin t.$$

$W_3(t) = -e^t \sin t - e^t \cos t.$

$y(t) = c_1 e^t + c_2 \cos t + c_3 \sin t +$

$e^t \int_0^t \frac{1}{2e^s \cos s} ds + \cos t \int_0^t \frac{-e^s \cos s + e^s \sin s}{2e^s \cos s} ds$

$+ \sin t \int_0^t \frac{-e^s \sin s - e^s \cos s}{2e^s \cos s} ds$

$= c_1 e^t + c_2 \cos t + c_3 \sin t$

$+ e^t \int_0^t \frac{1}{2e^s \cos s} ds - \frac{1}{2} \cos t \ln(\cos t)$

$- \frac{1}{2} t \cos t + \frac{1}{2} \sin t \ln(\cos t) - \frac{1}{2} t \sin t.$

4.3.5. $y = c_1 + c_2 t + c_3 e^{-2t} + c_4 e^{2t} - \frac{1}{3} e^t - \frac{1}{48} t^4 - \frac{1}{16} t^2$

18. $Y(t) = A e^t + (B_0 t + B_1) e^{-t} + t e^{-t} (C \cos t + D \sin t)$

$$y^{(4)} - 4y'' = t^2 + e^t$$

$$(D^4 - 4D^2)y = t^2 + e^t.$$

$$\Rightarrow (D^4 - 4D^2)D^3(D-1)y = 0$$

$$\Rightarrow D^5(D^2-4)(D-1)y = 0$$

$$\Rightarrow y = C_1 + C_2 t + C_3 t^2 + C_4 t^3 + C_5 t^4 + C_6 e^{2t} + C_7 e^{-2t} + C_8 e^t$$

in which, C_1, $C_2 t$, $C_6 e^{2t}$, $C_7 e^{-2t}$ solves $y^{(4)} - 4y'' = 0$.

Plug in $y = C_3 t^2 + C_4 t^3 + C_5 t^4 + C_8 e^t$ into the equation, then we have

$C_3 = -\frac{1}{18}$, $C_4 = 0$, $C_5 = -\frac{1}{48}$, $C_8 = -\frac{1}{3}$.

18. $y^{(4)}+2y'''+2y''=3e^{t}+2te^{-t}+e^{-t}\sin t$

$(D^4+2D^3+2D^2)y=3e^{t}+2te^{-t}+e^{t}\sin t$

$D^2(D^2+2D+2)\,y=3e^{t}+2te^{-t}+e^{t}\sin t$

annihilators of e^{t} is $D-1$

of te^{-t} is D^2+2D+1.

of $e^{t}\sin t$ is D^2-2D+2.

$\Rightarrow D^2(D^2+2D+2)(D-1)(D^2+2D+1)(D^2-2D+2)y=0$

$y=C_1+C_2t+C_3e^{t}\sin t+C_4e^{-t}\cos t+C_5e^{t}+C_6e^{-t}+C_7e^{-t}t+C_8e^{t}\sin t+C_9e^{t}\cos t$.

in which, C_1, C_2t, $C_3e^{t}\sin t+C_4e^{-t}\cos t$

solve $y^{(4)}+2y'''+2y''=0$

$\Rightarrow Y(t)=C_5e^{t}+C_6e^{-t}+C_7e^{-t}t+C_8e^{t}\sin t+C_9e^{t}\cos t$.